201c TOURNAMENT

Grade 6 and Below

PRACTICE PROBLEMS FOR 2019

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2016 Individual Event

No calculators are permitted during this tournament.
Time Limit: 30 minutes.

1. The sum of two consecutive multiples of 7 is 49. What is the greater of these numbers?

2. Terry walks 4/5 of the way from school to home in exactly 20 minutes. How long does it take for her to walk the rest of the way home?

3. A plane flies at 660 feet per second (ft/sec). How many miles per hour (mph) is this?
Note: 88 ft/sec is the same as 60 mph.

4. A number of identical wooden blocks are piled on each other to form a cube that is 12 centimeters on each side. Each block is a rectangular solid that is 2 centimeters by 3 centimeters by 6 centimeters. How many blocks are there?

5. One sandwich and a drink costs \$7.95. At the same prices, three sandwiches and a drink cost \$19.95. What is the price of a drink?

6. For every \$6 Emily has, Megan has \$4 and Grace has \$3. Olivia has the rest of the money. If the four girls have a total of \$61 *in dollar bills only*, what is the least number of dollar bills Olivia can have?

7. *ABCD* represents a four-digit number. For how many different odd numbers is *A* an even digit if no two digits are the same?

8. The total surface area of a cube is 96 square centimeters. The cube is enlarged so that each edge is increased by 6 centimeters. By how much is the total surface area of the resulting cube increased?

9. This addition at the right shows three 3-digit numbers. Every digit from 1 through 9 is used exactly one time. Each letter represents a digit. What ***3-digit number*** is represented by the sum *BD4*?

$$\begin{array}{r} 2\ C\ E \\ +\ A\ 3\ F \\ \hline B\ D\ 4 \end{array}$$

10. Of the 45 people in the school band, 17 do not ride to school and 22 are boys. If 15 of the girls ride to school, how many boys in the band do not ride to school?

Do not write in this space.

SCORE

Student Name ____________________________________

School and Team ______________________ Team # _______

2016 Individual Event: Answers

Write answers clearly. Each correct answer will receive one point.

1. ______________________________
2. ______________________________ **minutes**
3. ______________________________ **mph**
4. ______________________________ **blocks**
5. **$** ______________________________
6. ______________________________ **dollar bills**
7. ______________________________ **numbers**
8. ______________________________ **sq cm**
9. ______________________________
10. ______________________________ **boys**

2016 Team Event

No calculators are permitted during this tournament.
Time Limit: 20 minutes.

11. What is the tens digit in the product of $3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$?

12. I am 17 years younger than my aunt. If her age is 5 years less than three times mine, how old am I?

13. I have only coins but I cannot make change for a dollar, a half-dollar, a quarter, a dime, or a nickel. What is the greatest possible value of my coins?
(*Assume the half-dollar is the greatest coin in value.*)

14. The perimeter of a rectangle is 44 meters. The length is 4 meters greater than the width. What is the area of the rectangle?

15. If Ana's age is added to the square of Billy's age, the sum is 53. But if Billy's age is added to the square of Ana's age, the sum is 23. What is the sum of Ana's and Billy's ages?

16. A store discounts by always marking down each item by the same percent. The original price of one item was $32. The first discount resulted in a price of $24. The second discount resulted in a price of $18. What price resulted from the third discount?

17. Taylor has $5.00, all in dimes, quarters, or a combination of dimes and quarters. Which of the following cannot be the total number of coins that she has?
23, 29, 35, 37, 41, 50

18. A rectangle, a circle, and a regular pentagon are placed so that they overlap. No side of the rectangle coincides with a side of the pentagon. What is the greatest possible number of points of intersection?

19. What is the least counting number that is greater than 200 and has exactly three different factors?

20. The length, width, and semi-perimeter of a rectangle are each a prime number. The length is 13 cm more than eight times the width. What is the area of the rectangle?
(*The semi-perimeter is one-half of the perimeter.*)

Do not write in this space.

SCORE

School and Team ______________________________ Team # ________

Student Names ____

2016 Team Event: Answers

Write answers clearly. Each correct answer will receive one point.

11. ______________________________

12. ______________________________ **years**

13. **$** ______________________________

14. ______________________________ **sq m**

15. ______________________________ **years**

16. **$** ______________________________

17. ______________________________ **coins**

18. ______________________________ **points**

19. ______________________________

20. ______________________________ **sq cm**

SCORING ROOM KEY

2016

INDIVIDUAL Event: Answers		TEAM Event: Answers	
1.	28	11.	0
2.	5	12.	11
3.	450	13.	1.19
4.	48	14.	117
5.	1.95	15.	11
6.	9 or 9.00	16.	13.50
7.	1120	17.	37
8.	504	18.	26
9.	954	19.	289
10.	9	20.	58

INDIVIDUAL EVENT SOLUTIONS, 2016

ANSWERS: 1) **28** 2) **5** 3) **450** 4) **48** 5) **1.95**
6) **9** *or* **9.00** 7) **1120** 8) **504** 9) **954** 10) **9**

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1. ***METHOD 1:*** The first few multiples of 7 are 0, 7, 14, 21, 28, 35, 42, 49. The two multiples on the list that are consecutive and add up to 49 are 21 and 28. The greater of these numbers is $\boxed{28}$.

   METHOD 2: Half of 49 is between 24 and 25. The nearest two multiples of 7 are 21 and 28. They are consecutive and their sum is 49. The greater multiple of 7 is 28.

2. ***METHOD 1:*** Terry had walked $\frac{4}{5}$ of the way home. She has another $\frac{1}{5}$ of the way left. Because *one*-fifth is $\frac{1}{4}$ of *four*-fifths, $\frac{1}{4}$ of the 20 minutes remain to walk. It takes $\boxed{5}$ minutes to walk the rest of the way.

   METHOD 2: It takes Terry 20 minutes to walk $\frac{4}{5}$ of the way home. Therefore, it takes her 5 minutes to walk $\frac{1}{5}$ of the way and 25 minutes to walk $\frac{5}{5}$ (the whole distance). It takes her 25 – 20 = 5 minutes more to get home.

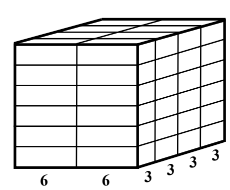
3. ***METHOD 1:*** Since 660 ÷ 88 = 7½, we must multiply 88 ft/sec by 7½ to get 660 ft/sec. Therefore we must multiply 60 mph by 7½ to get the speed of the plane in mph. The plane flies at $\boxed{450}$ mph.

   METHOD 2: Set up a proportion:

$$\frac{660}{88} = \frac{x}{60}$$

$$88x = 39{,}600$$

$$x = 450 \text{ mph}$$



4. ***METHOD 1:*** The volume of the 12 by 12 by 12 cube is 1728 cubic centimeters (cc). The volume of one 2 by 3 by 6 block is 36 cc. Then there are 1728 ÷ 36 = **48** blocks in the cube.

METHOD 2: Suppose every block in the cube is placed in the same direction as shown. The front of the cube shows 2 identical stacks that are 6 blocks high. That is 12 blocks. But the cube is four such sets of blocks deep. Therefore, there is a total of 12 × 4 = 48 blocks in all.

5. If two sandwiches are added to a \$7.95 order, they result in a \$19.95 order. This costs \$12 more, so each sandwich costs \$6. Therefore, one drink costs \$7.95 – \$6.00 = \$ $\boxed{1.95}$.

6. For every \$6 Emily has, Emily, Megan and Grace have a total of \$13. Thus the most they can have is 4 × 13 = \$52 and the least Olivia can have is 61 – 52 = \$ $\boxed{9}$.

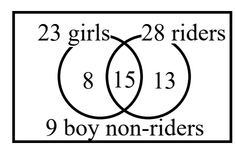
7. ABCD is an odd number, so D can be any of 5 digits: 1, 3, 5, 7, or 9. Since A is even and ABCD has four digits, A can be any of four digits: 2, 4, 6, or 8, but not 0. This leaves any of eight digits for B and any of seven for C. Thus, 4 × 8 × 7 × 5 = $\boxed{1120}$ different four-digit numbers are possible.

8. There are six congruent surfaces to a cube. Before the cube was enlarged, the area of one surface was 96 ÷ 6 = 16 sq cm. Each surface is a square so the length of one edge was 4 cm. After the cube is enlarged, the length of one edge is 10 cm. Then the area of one surface is 100 sq cm and the total surface area is 600 sq cm. The increase is 600 – 96 = $\boxed{504}$ sq cm.

9. The missing digits are 1, 5, 6, 7, 8, and 9. Thus, E + F can only be 5 + 9 or 6 + 8 in some order. Either way, the tens column is increased by 1. Thus C + 4 ends in D; so C,D in that order is 1,5 or 5,9 or 7,1. However, the 5 can only be used once. If C is 5 or 7, then the hundreds column is increased by 1 and 3 + A = B. Thus A,B = 5,8 or 6,9. But if C is 1, the hundreds column is not increased and 2 + A = B, so that A,B is 5,7 or 6,8 or 7,9. Of all these choices, only 216 + 738 = 954 or 218 + 736 = 954 uses all nine of the digits, each once. Either way, the sum is $\boxed{954}$.

$$\begin{array}{r} 2\ C\ E \\ +\ A\ 3\ F \\ \hline B\ D\ 4 \end{array}$$

10. ***METHOD 1:*** Because 15 girls do ride to school, consider two overlapping groups: the girls and the riders. There are 45 – 22 = 23 girls and 45 – 17 = 28 riders, with 15 in both groups. How many belong to only one group? 23 – 15 = 8 girls are not riders and 28 – 15 = 13 riders are not girls (they are boys). Since 13 of the 22 boys ride to school, $\boxed{9}$ boys do not ride to school.



METHOD 2: As in method 1, there are 23 girls and 28 riders, with 15 people being counted twice since they are in both groups. This accounts for a total of 23 + 28 – 15 = 36 people. The remaining 45 – 36 = 9 people are neither girls nor riders, so there are 9 boys who do not ride to school.

TEAM EVENT SOLUTIONS, 2016

ANSWERS: 1) **0** 2) **11** 3) **1.19** 4) **117** 5) **11**
6) **13.50** 7) **37** 8) **26** 9) **289** 10) **58**

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11. Since 5 × 6 = 30 and 30 × 10 = 300, the final product is a multiple of 300. The tens digit is [0].

12. ***METHOD 1:*** My aunt is more than 17 years old. So I must be at least (18 + 5) ÷3 = almost 8 years old. In the table below, the first row lists my possible ages, the second row triples my age and then subtracts 5 years and the third row lists the difference in years between our ages.

My age	8	9	10	**11**
My aunt's age	24 – 5 = 19	22	25	**28**
Difference	11	13	15	**17**

Only when I am [11] years old is she 17 years older than me.

METHOD 2: Let x represent my age in years. Then $3x - 5$ represents my aunt's age.

Equation: Her age minus my age = 17 years: $(3x - 5) - x = 17$

Combine like terms: $2x - 5 = 17$

Add 5 to each side of the equation: $2x = 22$

Divide both sides of the equation by 2: $x = 11$ I am 11 years old.

13. Start with the least coin and add the others one at a time. I cannot have more than 4 pennies or else I could make change for a nickel. If I then also use four dimes, one quarter and one half-dollar, I would not be able to make change for any of the amounts listed. The greatest possible value is [$1.19].

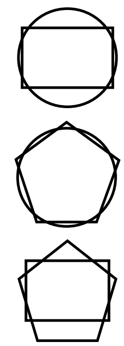
14. The semi-perimeter is half the perimeter because it is the sum of one length and one width. Then the semi-perimeter is 22 m. Since the length is 4 m greater than the width, we need two numbers that add to 22 and subtract to 4. The rectangle is 13 m long and 9 m wide. The area is [117] sq m.

15. The square of Billy's age is less than 53, namely 49, 36, 25, 16, 9, 4, or 1. Thus, Billy is 7 years old or less. If Billy were 7, Ana would be 53 – 49 = 4. Similarly, if Billy were 6, 5, or 4, Ana would be 53 – 36 = 17, 53 – 25 = 28, or 53 – 16 = 37. On the other hand, the square of Ana's age is less than 23. Of 4, 17, 28, and 37, only the square of 4 is less than 23. The sum of Ana's and Billy's ages is 4 + 7 = [11] years.

16. After applying the first discount, $24 is $\frac{24}{32}$ of $32. The second price is $\frac{3}{4}$ or 75% of the first price. (The discount itself is 25% or $\frac{1}{4}$.) Applying the second discount, $\frac{3}{4}$ of $24 is $18, as given. Then applying the third discount, $\frac{3}{4}$ of $18 is [$13.50].

17. Start with \$5 all in quarters, trade quarters for dimes and examine the total numbers of coins. If Taylor has 20 quarters, she would have 0 dimes, for a total of 20 coins. Since two quarters have the same value as five dimes, Taylor can have 20, 18, 16, …, 4, 2, 0 quarters. Consequently, she would have 0, 5, 10, …, 50 dimes respectively. Then she would have a total of 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50 coins. Thus, she cannot have $\boxed{37}$ coins.

18. Draw the figures in pairs, as shown. Make sure that all three figures do not pass through any one point. In each case trace one figure and think of it as entering and then leaving the other figure; this means there must be an even number of points of intersection.



The circle leaves and then reenters each of the four sides of the rectangle, so they have 8 points in common.

The circle leaves and then reenters each of the five sides of the pentagon, so they have 10 points in common.

The pentagon leaves and then reenters each of the four sides of the rectangle, so they have 8 points in common (rotating the rectangle would not add two more points).Thus, the greatest possible number of points of intersection is $8 + 10 + 8 = \boxed{26}$.

19. Examine simpler numbers: 1, 2, 3, 4, … The number 4 has just three factors: 1 and 4 itself and 2 because $2 \times 2 = 4$. Similarly, the number 9 has three factors: 1 and 9 itself and 3 because $3 \times 3 = 9$. So two factors are 1 and the number itself, and the third factor is the square root of our number. Thus, we are looking for a perfect square number. But 16 has five factors: 1, 2, 4, 8, and 16, because 4, the square root of 16, is not a prime number. Check the squares of prime numbers: the factors of 25 are 1 and 25, and 5. Also, the factors of 49 are 1 and 49, and 7. The following table lists the squares of all the prime numbers less than 300.

Prime number	2	3	5	7	11	13	**17**
Square of the prime	4	9	25	49	121	169	**289**
Factors	1, 2, 4	1, 3, 9	1, 5, 25	1, 7, 49	1, 11, 121	1, 13, 169	**1, 17, 289**

The least number which is greater than 200 and has just three factors is $\boxed{289}$.

20. Every prime number except 2 is odd. An odd number is the sum of an odd number and an even number. Thus, the width is 2 cm, and both the length and the semi-perimeter are consecutive *odd* prime numbers, since the semi-perimeter is the sum of one length and one width. Then the length is $8 \times 2 + 13 = 29$ cm and the semi-perimeter is $2 + 29 = 31$ cm; both 29 and 31 are prime numbers. The area of the rectangle is $2 \times 29 = \boxed{58}$ sq cm.

TIEBREAKER #1

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

Three different members of the set {4, 5, 6, 7, 8, 9} are chosen at random and added. How many different sums are possible?

Answer ________________ sums

TIEBREAKER #2

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

Ten days from a Sunday is a Wednesday. What day of the week is 100 days from a Wednesday?

Answer _____________________

TIEBREAKER #3

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

In the addition below, what four-digit number is represented by ABCD?

$$\begin{array}{r} B\ 5\ 6 \\ 5\ C\ 9 \\ +\ 7\ 4\ D \\ \hline A\ 0\ 0\ 0 \end{array}$$

Answer ABCD = ________________

TIEBREAKER #4

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

How many different whole numbers can be formed from the digits 9, 7, 5, and 3?

Answer ______________ numbers

TIEBREAKER #5

Time Limit: 5 minutes.
No calculators are permitted.

Name ________________________________ Team _________________________

The number 5,472,B68 is a multiple of 9. What digit does B represent?

Answer _____________________

2016 TIEBREAKERS

Questions are given one at a time. Winning places are awarded in the order that correct answers are submitted. Incorrect answers result in elimination. No calculators are permitted during this tournament. Time limit: 5 minutes per question.

T1. Three different members of the set {4, 5, 6, 7, 8, 9} are chosen at random and added. How many different sums are possible?

T2. Ten days from a Sunday is a Wednesday. What day of the week is 100 days from a Wednesday?

T3. In the addition at the right, what four-digit number is represented by ABCD?

	B	5	6
	5	C	9
+	7	4	D
A	0	0	0

T4. How many different whole numbers can be formed from the digits 9, 7, 5, and 3?

T5. The number 95,472,B68 is a multiple of 9. What digit does B represent?

SOLUTIONS

ANSWERS: T1) **10** T2) **Friday** T3) **2695** T4) **24** T5) **4**

T1. The sum is at least 15 and at most 24, and can be any whole number from 15 to 24 inclusive. Hence, $\boxed{10}$ sums are possible.

T2. ***METHOD 1:*** Form a calendar and count the days starting at a Thursday. The 100th day is a $\boxed{\text{Friday}}$.

METHOD 2: Divide 100 by 7. There are 14 weeks and then two extra days. The two extra days are Thursday and Friday.

T3. In the ones column, 6 + 9 = 15. Then to get a sum that ends in 0, D must be a 5 and 2 will be carried.
In the tens column, 2 + 5 + 4 = 11. For a sum that ends in 0, C must be a 9 and 2 will be carried.
In the hundreds column, 2 + 5 + 7 = 14. For a sum that ends in 0, B must be a 6 and 2 will be carried.
Then A will be a 2. The four-digit number is $\boxed{2695}$.

T4. ***METHOD 1:*** Any of the 4 digits can occupy the thousands place. For each choice, any of the remaining 3 digits can occupy the hundreds place. For each of the 4 × 3 = 12 arrangements, either of the remaining 2 digits can occupy the tens place. For each of the 4 × 3 × 2 = 24 arrangements, the remaining digit must occupy the units place. There are a total of $\boxed{24}$ four digit numbers that can be formed.

METHOD 2: List them:

3579	5379	7359	9357
3597	5397	7395	9375
3759	5739	7539	9537
3795	5793	7593	9573
3957	5937	7935	9735
3975	5973	7953	9753

T5. The sum of the digits must be a multiple of 9: 5 + 4 + 7+ 2 + B + 6 + 8 = 32 + B. Then B = $\boxed{4}$. Note: It is faster to 'cross out nines' (5 + 4 and 7 + 2) since each adds up to 9. Then just test B + 6 + 8 for divisibility by 9.